Today you will:

- Solve absolute value equations
- Solve equations involving two absolute values
- Identify special solutions of absolute value equations

Absolute value of x is the magnitude of the number ignoring its sign

$|\pm x| = x$ ~~***NOT*** $|x| = \pm x$~~

Examples:

|3|=3

$|-3| = 3$

The result of an absolute value is never negative …

In other words you can never have something like $|x| = -3$

Properties of Absolute Value

1. $|a| \geq 0$

2. $|-a| = |a|$

3. $|ab| = |a||b|$

4. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $b \neq 0$

Solving Absolute Value Equations

To solve $|ax + b| = c \; when \; c \geq 0$, solve the two related linear functions

$$ax + b = c$$

or

$$ax + b = -c$$

Remember, when c < 0, the absolute value equation $|ax + b| = c$ has **no solution** because absolute value always indicates a number that is not negative.

Solve each equation. Graph the solutions, if possible.

a. $|x - 4| = 6$ **b.** $|3x + 1| = -5$

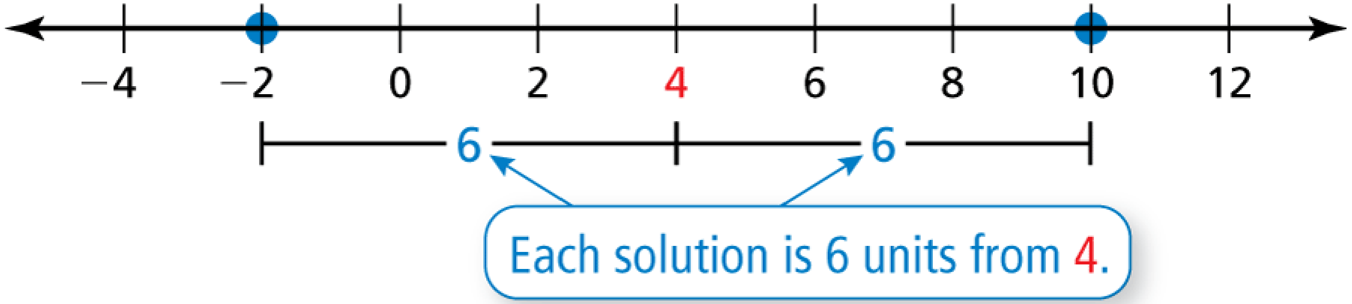
SOLUTION

a. Write the two related linear equations for $|x - 4| = 6$. Then solve.

$x - 4 = 6$ *or* $x - 4 = -6$ Write related linear equations.

$x = 10$ $x = -2$ Add 4 to each side.

The solutions are $x = 10$ and $x = -2$.



Property of Absolute Value

b. The absolute value of an expression must be greater than or equal to 0. The expression $|3x + 1|$ cannot equal -5.

So, the equation has no solution.

Solve $|3x + 9| - 10 = -4$

SOLUTION

First isolate the absolute value expression on one side of the equation.

ANOTHER WAY

Using the product property of absolute value, $|ab| = |a|\,|b|$, you could rewrite the equation as

$$3|x + 3| - 10 = -4$$

and then solve.

$\lvert 3x + 9\rvert - 10 = -4$	Write the equation.
$\lvert 3x + 9\rvert = 6$	Add 10 to each side.

Now write two related linear equations for $|3x + 9| = 6$. Then solve.

$3x + 9 = 6$	*or*	$3x + 9 = -6$	Write related linear equations.
$3x = -3$		$3x = -15$	Subtract 9 from each side.
$x = -1$		$x = -5$	Divide each side by 3.

The solutions are $x = -1$ and $x = -5$.

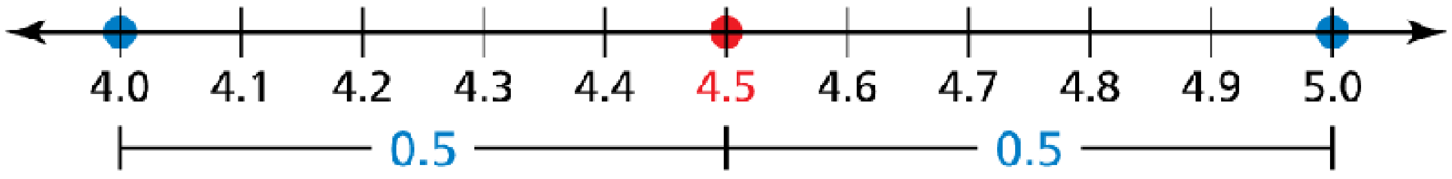
In a cheerleading competition, the minimum length of a routine is 4 minutes. The maximum length of a routine is 5 minutes. Write an absolute value equation that represents the minimum and maximum lengths.

SOLUTION

1. **Understand the Problem** You know the minimum and maximum lengths. You are asked to write an absolute value equation that represents these lengths.

2. **Make a Plan** Consider the minimum and maximum lengths as solutions to an absolute value equation. Use a number line to find the halfway point between the solutions. Then use the halfway point and the distance to each solution to write an absolute value equation.

3. Solve the Problem



halfway point

distance from halfway point

$$|x - 4.5| = 0.5$$

The equation is $|x - 4.5| = 0.5$

4. Look Back To check that your equation is reasonable, substitute the minimum and maximum lengths into the equation and simplify.

Minimum	**Maximum**
$|4 - 4.5| = 0.5$ ✓	$|5 - 4.5| = 0.5$ ✓

Solve (a) $|3x - 4| = |x|$ and (b) $|4x - 10| = 2|3x + 1|$.

SOLUTION

a. Write the two related linear equations for $|3x - 4| = |x|$. Then solve.

$$3x - 4 = x \quad \text{or} \quad 3x - 4 = -x$$

$$\underline{-x} \quad \underline{-x} \qquad \underline{+x} \quad \underline{+x}$$

$$2x - 4 = 0 \qquad 4x - 4 = 0$$

$$\underline{+4} \quad \underline{+4} \qquad \underline{+4} \quad \underline{+4}$$

$$2x = 4 \qquad 4x = 4$$

$$\frac{2x}{2} = \frac{4}{2} \qquad \frac{4x}{4} = \frac{4}{4}$$

$$x = 2 \qquad x = 1$$

The solutions are $x = 2$ and $x = 1$.

Check

$$|3x - 4| = |x|$$

$$|3(2) - 4| \stackrel{?}{=} |2|$$

$$|2| \stackrel{?}{=} |2|$$

$$2 = 2 \ \checkmark$$

$$|3x - 4| = |x|$$

$$|3(1) - 4| \stackrel{?}{=} |1|$$

$$|-1| \stackrel{?}{=} |1|$$

$$1 = 1 \ \checkmark$$

b. Write the two related linear equations for $|4x - 10| = 2|3x + 1|$. Then solve.

$$4x - 10 = 2(3x + 1) \quad \text{or} \quad 4x - 10 = 2[-(3x + 1)]$$

$$4x - 10 = 6x + 2$$

$$\underline{-6x} \qquad \underline{-6x}$$

$$-2x - 10 = 2$$

$$\underline{+10} \qquad \underline{+10}$$

$$-2x = 12$$

$$\frac{-2x}{-2} = \frac{12}{-2}$$

$$x = -6$$

$$4x - 10 = 2(-3x - 1)$$

$$4x - 10 = -6x - 2$$

$$\underline{+6x} \qquad \underline{+6x}$$

$$10x - 10 = -2$$

$$\underline{+10} \qquad \underline{+10}$$

$$10x = 8$$

$$\frac{10x}{10} = \frac{8}{10}$$

$$x = 0.8$$

The solutions are $x = -6$ and $x = 0.8$.

Solve $|2x + 12| = 4x$. Check your solutions.

SOLUTION

Write the two related linear equations for $|2x + 12| = 4x$. Then solve.

$2x + 12 = 4x$	*or*	$2x + 12 = -4x$	Write related linear equations.
$12 = 2x$		$12 = -6x$	Subtract $2x$ from each side.
$6 = x$		$-2 = x$	Solve for x.

Check the apparent solutions to see if either is extraneous.

Check

$|2x + 12| = 4x$

$|2(6) + 12| \stackrel{?}{=} 4(6)$

$|24| \stackrel{?}{=} 24$

$24 = 24$ ✓

$|2x + 12| = 4x$

$|2(-2) + 12| \stackrel{?}{=} 4(-2)$

$|8| \stackrel{?}{=} -8$

$8 \neq -8$ ✗

The solution is $x = 6$. Reject $x = -2$ because it is extraneous.

When solving equations of the form $|ax + b| = |cx + d|$, it is possible that one of the related linear equations will not have a solution.

Solve $|x + 5| = |x + 11|$.

SOLUTION

By equating the expression $x + 5$ and the opposite of $x + 11$, you obtain

$x + 5 = -(x + 11)$	Write related linear equation.
$x + 5 = -x - 11$	Distributive Property
$2x + 5 = -11$	Add x to each side.
$2x = -16$	Subtract 5 from each side.
$x = -8$	Divide each side by 2.

However, by equating the expressions $x + 5$ and $x + 11$, you obtain

$$x + 5 = x + 11$$

$$x + 5 = x + 11 \quad \text{Write related linear equation.}$$

$$x = x + 6 \quad \text{Subtract 5 from each side.}$$

$$0 = 6 \;\; ✗ \quad \text{Subtract } x \text{ from each side.}$$

which is a false statement. So, the original equation has only one solution.

REMEMBER

Always check your solutions in the original equation to make sure they are not extraneous.

The solution is $x = -8$.